THE FLOW INDUCTION PROCESS OF COMPRESSORS DESIGNED TO ACCEPT SUPERSONIC INFLOW VELOCITY

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ABSTRACT

The use of a compressor capable of accepting supersonic inflow velocity has been advocated in the past as a means of eliminating the relatively long subsonic diffuser portion of a supersonic inlet and as a means of ensuring inlet-compressor airflow match at supersonic speeds without the need for variable inlet geometry. A detailed analysis of the compatibility of the inlet and a compressor of this type has not appeared in the literature, nor has the flow pattern in the forward portion of the compressor element been examined. This paper presents a study using a simplified two-dimensional cascade approach to determine the effects of Mach number, cone angle, rotational speed and blade angle upon conditions limiting true supersonic entry, relative Mach number, and shock loss. The interaction between suction surface oblique shock and boundary layer on the following blade is examined using the Chapman laminar leading-edge separation criteria and permissible shock flow patterns are diagramed. A theory for variable-pitch supersonic guide vanes is developed and the effect of guide vanes upon compressor inflow condition is presented.

INTRODUCTION

The supersonic compressor has been proposed in the past for use as an element of a gas turbine engine operating at supersonic flight speeds because of its ability to accept supersonic axial inflow velocity (refs. 1 to 3). The elimination of both the subsonic portion of an axisymmetric supersonic inlet diffuser, and the requirement for variable inlet geometry together with the

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high-stage-pressure-ratio potential of the supersonic compressor can result in a very significant reduction of propulsion system weight. Whether or not the weight reduction of the powerplant will result in a lower take-off gross-weight aircraft designed for a given long-range supersonic cruise segment will depend upon the cruise specific impulse which in turn depends upon the overall compression process efficiency of the supersonic inflow engine as compared to that of the conventional subsonic design. On the other hand, the lower weight of the supersonic inflow engine although utilizing a relatively inefficient compressor may show a significant advantage in take-off gross weight when used for accelerating a hypersonic cruise aircraft to its ramjet engine take-over Mach number since the propulsion efficiency is relatively low in any case in which maximum afterburning thrust is employed.

The present paper, which is basically an amplification of the analysis presented in reference 3, examines the flow conditions in the leading-edge portion of a supersonic compressor for flight Mach numbers up to 3 by means of a two-dimensional cascade approach as the design parameters of blade speed, inlet diffusion, and blade suction surface angle are varied. As pointed out in reference 3, not only is the leading-edge shock loss chargeable to compressor efficiency but the inclination of the oblique shock will determine whether or not the inlet can operate supersonically. Thus the engine cycle analysis of reference 4 which indicates the superiority of the supersonic axial inflow compressor with respect to a conventional multistage compressor for Mach number 3 operation must be tempered by the fact that the flow induction loss of the compressor was not examined.

The results of the analysis are equally applicable to any type of supersonic compressor whether of the single-stage impulse, the fan stage of a turbofan, or the first stage of a multistage compressor. Because the type and design pressure ratio of the compressor are not specified, and because of a lack of experimental performance data available on supersonic inflow operation of supersonic compressors (with the exception of data in ref. 3), no attempt is made to predict the overall compression efficiency. However, where appropriate, the effect of the calculated compressor flow conditions upon the propulsion system performance will be noted.

SYMBOLS

A	area
a	speed of sound
B	defined parameter, Appendix B
C	defined parameter, Appendix B
c	guide vane chord
C_F	average skin-friction coefficient
C_D	drag coefficient
C_L	lift coefficient
C_N	normal-force coefficient
D	drag force
G	guide vane pitch distance
L	lift force
M	Mach number
p	pressure
S	entropy
T	absolute temperature
t	guide vane thickness
U	rotational velocity
V	airflow velocity
Y	distance in circumferential direction
α	guide vane angle of attack
β	airflow angle relative to axial direction
γ	ratio of specific heats

δ leading-edge wedge angle

η cone semivertex angle

ρ density

σ solidity

σ_M maximum solidity without mutual interference between guide vane blades

ϕ shock wave angle relative to axial direction

ψ surface angle relative to axial direction

Subscripts:

Stations -

0 - free stream

1 - upstream of strut or guide vanes

2 - upstream of rotor

3 - behind compressor leading-edge oblique shock, figure 8

4 - behind system of oblique shocks, figure 8

a axial

D design condition

E location in flow field, figure 8

id ideal (loss free)

P location in flow field, figure 8

R relative conditions

S suction surface

T stagnation condition

u tangential

GENERAL ASSUMPTIONS

To examine the flow induction process of the supersonic inlet-compressor combination, several simplifying assumptions are employed. The inflow process to the compressor is considered in a two-dimensional cascade-type approach utilizing the arrangement depicted in figures 1 and 2. The supersonic flow on the conical spike is turned parallel to the axis in a constant area duct from the cowl lip station C to the compressor inlet station 2 in a loss-free manner in the absence of vanes. Further, it is assumed that the axial Mach number entering the compressor is equal to the cone surface Mach number. This, of course, violates the weight flow continuity equation as there is a Mach number variation in the conical flow field; however, the deviation and its effect upon inflow conditions under most conditions is small. This then permits a consistent approach to the determination of the axial inlet Mach number as inlet diffusion (as represented by the cone angle) and flight Mach numbers are varied; a cone-isentropic-ramp external compression inlet would yield essentially similar results if the annular area were equal to that of the conical inlet.

The annular walls are assumed to be distorted in the region of the support struts or guide vanes such that the effect of blockage associated with strut or vane thickness is eliminated. This is done to prevent any internal inlet area contraction which increases the Mach number at which the inlet will start. The inlet is assumed to be part of an isolated nacelle operating in the stratosphere and out of the interference field associated with the flow about an aircraft. In addition, the air is treated as a perfect inviscid gas.

The "entrance region" on the suction surface of the compressor is assumed to be straight. (The "entrance region" as defined in reference 1 is that portion of the blade which can generate upstream traveling waves at supersonic relative flow conditions and so affect the inflow velocity.) The blade leading edge is sharp and its wedge angle δ is small enough to permit shock attachment at the design sea-level operating condition. The relative air inlet angle β_{2R} for subsonic or sonic axial inflow conditions must therefore be equal to the slope of the "entrance region" (ref. 1). Thus the entrance region angle can be characterized by a design sea-level static axial inlet Mach number $M_{a,D}$.

In several instances, the effects of varying these basic assumptions are examined in light of experience and practical considerations and are specifically noted where they occur.

INLET-COMPRESSOR FLOW ANALYSIS

Limiting Conditions

The necessary requirement for the existence of supersonic axial inflow velocity to the compressor during supersonic flight is that the oblique shock wave generated in the relative flow field by the suction surface of the blade "entrance region" lies in or behind the plane of the leading edges (ref. 3). If the calculated shock wave angle, measured from the axial direction, is greater than 90°, the shock cannot exist at the leading edge of an infinite cascade. As a consequence, the subsonic axial inflow requirement of the compressor forces a bow shock to be positioned ahead of the cowl lip creating additional spillage drag due to subcritical inlet operation. To avoid the additional drag penalty associated with the subcritical spillage, it is desirable to maintain supersonic axial inlet velocity into the compressor.

Since the oblique shock wave angle is a function only of the relative inlet Mach number $M_{2,R}$, the relative air inlet angle $\beta_{2,R}$, and the surface angle ψ_s, it is a simple matter to calculate the relationship of these parameters such that the limiting conditions for supersonic inflow can be determined. Appendix A presents the computational procedure and figure 3, the results of such calculations. The nearly vertical transverse curve is the locus of conditions for which the oblique shock spans the cascade leading edge and supersonic turning is a maximum. To the left of this curve the shock lies in the passage, and flow deflection everywhere is still a maximum. For a given surface angle and Mach number, any reduction in relative inlet air angle below the limiting value will cause shock detachment and position a bow shock ahead of the cowl lip as the waves from adjacent blades cannot cross.

To the right of the transverse curve the oblique shock is coincident with the cascade leading edge. If for a given surface angle and relative air inlet angle the relative Mach number is greater than that for the limiting curve, the shock will fall inside the passage and supersonic axial inflow is assured. The same is true, if for a given surface angle and relative inlet Mach number, the relative inlet air angle is greater than that for the limiting curve. The lower inclined straight line is the limiting condition for sonic axial inflow velocity for which the leading-edge shock degenerates to a Mach wave.

Typical Operating Characteristics

For a given fixed-geometry inlet-compressor combination, it is of interest to determine the flow pattern in the leading-edge region of the compressor and whether or not the limiting conditions are exceeded as the flight Mach number is varied. Since the relative inlet air angle and Mach number are functions of rotational speed U and the cone angle η (characterizing the degree of inlet diffusion), and the blade suction surface angle is characterized by $M_{a,D}$, it is possible to examine the inflow conditions in terms of variations in these parameters as is done in figures 4 to 6. Perturbations are made about a basic configuration for which $U = 1600$ feet per second, $\eta = 20^{\circ}$, and $M_{a,D} = 1.0$. (Reference 3 suggested that for $M_{a,D} = 1.0$ there would be no subcritical spillage; however, this is true only for the condition of constant free-stream stagnation temperature.)

<u>Effect of rotational velocity variation</u>.- Figure 4 presents the effects of variation in design rotational velocity upon inflow conditions. As noted previously, the speed is held constant at its design value throughout the Mach number range so that the effects of rpm changes during flight should not be inferred from the results presented in figure 4. The limiting condition, i.e., shock wave angle of 90°, occurs for all rotational speeds at a Mach number of 1.35 which in this case ($\eta = 20^{\circ}$) produces sonic velocity on the cone. The fact that all rotational speeds obtain the limiting condition at

the same flight Mach number indicates that a compressor will permit supersonic inflow to start at all radii at once. For Mach numbers from 1.35 to 3.0, the shock wave will fall within the blade passage with greater inclination for the higher rotational speeds.

The stagnation pressure recovery parameter references the stagnation pressure relative to the rotor behind the leading-edge shock (including the conical shock loss) to that of the ideal loss-free value. As can be seen, the relative stagnation pressure recovery decreases with increasing design rotational velocity. This is a consequence of the higher relative inlet Mach number and greater inclination of the shock wave angle ϕ and thus a higher Mach number component normal to the oblique shock. The relative inlet Mach number $M_{2,R}$ increases with flight Mach number and rotational velocity when supercritical flow exists at the cowl lip as would be expected. The Mach number behind the oblique shock $M_{3,R}$ decreases with flight Mach number as the oblique shock becomes stronger, as evidenced by the difference between $M_{2,R}$ and $M_{3,R}$. It should be noted that $M_{3,R}$ is essentially equal to the average passage inlet Mach number because of the high shock wave angle. At a flight Mach number of 1.35, the limiting condition is reached and a small discontinuity in relative inlet Mach number occurs as the shock moves from the cowl lip to the compressor. Between the Mach numbers of 1.29 and 1.35, the axial inlet Mach number is subsonic as a result of exceeding the limiting conditions. In this range of Mach number a spillage of less than 0.1 percent of the annular cowl choked flow is required. At Mach numbers less than 1.29, the rotor will demand full choking at the cowl lip with subsequent supersonic expansion about the leading edge yielding a passage inlet Mach number $M_{3,R}$ greater than the relative inlet Mach number as shown.

From the above it would appear desirable to operate at the lower rotational velocity because of the lower Mach number and resultant lower losses. However, this will restrict the compressor stage pressure ratio to some degree or else require greater flow turning in the rotor and consequently a higher downstream stator entering Mach number to achieve the same pressure ratio as that for a higher rotational speed design.

Effect of blade suction surface angle.- The suction surface angles were chosen such that at sea-level standard conditions the design axial inlet Mach number in a constant area passage was 0.8 and 1.0. In addition, the blade angle calculated for sonic inflow was arbitrarily reduced by 2° for the third variation chosen for study. For this latter case the inflow velocity remains sonic but the flow must expand about the leading edge to a higher relative surface Mach number.

The required surface inclination for a design axial inlet Mach number of 0.8 will prevent supersonic inflow until a flight Mach number of 2.0 is reached (where $\phi = 90^{\circ}$, fig. 5) and then again encounters the limiting condition at a Mach number of 2.9. As a consequence of the subcritical operation at either end of the supersonic flight spectrum and the nearly normal inclination of the oblique shock wave in between, the relative pressure recovery parameter is essentially equal to the ideal (inviscid) pressure recovery parameter of a 20° conical all-external compression inlet. The discontinuous variation of the inlet relative Mach number at the two limiting conditions for the 0.8 design is due to shock location and does not result in any discontinuity in compressor operation as indicated by the smooth variation of blade surface Mach number.

Decreasing the blade surface angle 2° as compared to the $M_{a,D} = 1.0$ case results in the disappearance of the oblique shock wave at a flight Mach number of 1.4 as the inlet relative flow direction is parallel to the blade suction surface. Below a flight Mach number of 1.4 the suction surface Mach number rises rapidly as supersonic expansions occur about the leading edge and the Mach number in the blade passage is considerably greater than the sea-level design value. Although the relative stagnation pressure recovery is higher than that for the $M_{a,D} = 1.0$ case, the higher surface Mach numbers associated with the lower surface angle would indicate higher diffusion losses for a shock-in-rotor type compressor. The same result is not necessarily true for the so-called impulse type. The net conclusion appears then that the compressor should be designed to induce sonic inflow velocity at sea-level standard conditions to avoid the penalties in thrust performance caused by the additional spillage drag associated with subcritical operation and the low pressure recovery of supersonic compressors designed for subsonic inflow velocity.

Effect of conical angle.- The effect of inlet external diffusion, as represented by cone angle, is shown in figure 6. Increasing the cone angle increases the flight Mach number at which the limiting condition is reached. The limiting condition for each cone, however, occurs at a flight Mach number only slightly greater than that for which the flow on the cone is sonic. Consequently, the added inlet spillage drag as a result of subsonic entry flow to the compressor would be small. The relative stagnation pressure ratio shows little variation with cone angle; the 20° conical inlet has slightly higher recovery. The lower conical shock loss produced by the smaller angle cone is more than compensated for by the higher oblique shock loss resulting from the higher inflow Mach number and flow deflection angle. The choice of conical angle to use in conjunction with the supersonic inflow compressor will depend upon factors other than the desire to maximize the relative stagnation pressure recovery.

Flow capacity.- In the previous sections, it was noted that a compressor designed for sonic inflow velocity at sea-level standard conditions will accept the maximum flow the inlet can deliver over a wide Mach number range with the exception of minute spillage at low supersonic flight Mach numbers. It is of interest to compare the airflow requirements of a conventional turbojet with the air inflow to a supersonic compressor. The parameter used for comparison consists of the ratio of the free-stream tube area containing the inlet flow to the cowl-lip projected area. On the left of figure 7, the area ratio variation of a typical turbojet is compared with that of three fixed geometry conical inlets operating supercritically all of which are designed for shock-on-lip at a Mach number of 3. The fixed geometry inlet spillage from the point at which sonic velocity is attained on the cone to a flight Mach number of 3 was obtained from reference 5. The turbojet schedule shown is for an engine with a design compressor ratio of 9 and is presumed to be operating with a variable geometry inlet which has a pressure recovery schedule given by military specification 5008B. The free-stream tube area of the conical inlets is appreciably greater than that demanded by the turbojet, as a consequence for equal conical angles, the associated supercritical spillage drag of the fixed geometry inlet is lower.

On the right of figure 7, the capture area ratio for a 20° cone in conjunction with two compressors designed for sonic and subsonic axial inlet Mach numbers is compared to that of the turbojet. Although the subsonic axial inflow design does not permit inlet starting until a Mach number of 2.0 is reached, it can still induce a higher flow rate than a conventional engine. At a flight Mach number of 1, there is little difference in the flow rate for the three cases shown, but in the region of minimum thrust minus drag, which generally occurs near Mach number 1.3, 13 percent more flow can be channeled into the sonic inflow design compressor as compared with a conventional design. An increase of 30 percent is possible with the 15° conical inlet. As a consequence, the thrust potential of an engine employing the supersonic inflow compressor at the critical accelerating point in the flight trajectory can be increased a like amount. The use of variable geometry components in the engine may be required to achieve this increased thrust potential in order to match the permissible engine airflow to that which the inlet-supersonic compressor combination can provide over the entire Mach number range.

Shock boundary-layer interactions can and do alter the flow patterns calculated by means of the inviscid supersonic flow theory, particularly in regions where large positive pressure gradients exist. In the previous analysis it was noted that the inclination of the leading-edge oblique shock is large and thus impinges near the pressure surface leading edge of the following blade where the boundary layer is most probably laminar. Since the boundary layer in most cases cannot sustain the pressure rise associated with shock reflection, the flow must separate from the surface and possibly affect the upstream inflow pattern to the extent that the inlet will be forced to spill.

To investigate this problem a specific example is examined in figure 8 with inflow conditions as shown. A Prandtl-Meyer expansion about the leading edge for the case labeled inviscid, shown on the right, is followed some distance downstream by the shock reflection. If the Reynolds number, based upon local flow conditions behind the expansion fan and the distance from the leading edge to the point of shock impingement, is greater than $4.52 \times 10^{(4 + 0.345M)}$, where M is the local Mach number, then the influence of any flow separation, if any occurs, will not be felt at the leading edge whether the flow is inviscid or not. (See fig. 24 of ref. 6.) If the Reynolds number is less than the limiting value, then in all probability the flow will separate at the leading edge. The laminar leading-edge separation theory of reference 6 can be used with the following provision to calculate the resulting flow diagram as depicted on the left side of figure 8. Namely, where the leading-edge oblique shock from the suction surface intersects the laminar separated boundary layer it is reflected as a Prandtl-Meyer expansion fan to keep the pressure constant in the separated region and in so doing turns the separated flow toward the surface. The permissible pressure in the separated region can be found and consequently the entire flow field diagramed by the necessary requirement (ref. 6) that the Mach number after reattachment M_p is 81 percent of the value of the Mach number behind the expansion M_E provided the flow remains laminar to reattachment. If boundary-layer transition occurs prior to reattachment, the pressure in the separated zone can be greater; however, the flow-field calculation is not amenable to analytic analysis.

In the typical example of figure 8, where downstream influences of wall curvature have been neglected, it can be seen that the Mach number and stagnation pressure recovery behind the reflected shock system are almost identical whether or not flow separation exists. Although the theory and its experimental verification presented in reference 6 of the leading-edge separation were based upon a concave corner model, there is little reason to doubt that its application to the case of an incident shock is not valid. For example, operation of supersonic inlets above the shock-on-lip design Mach number for which the internal cowl lip separation phenomena must be similar to that depicted above have been shown to exhibit no discontinuities in performance. At flight Mach numbers of 2.4 and above, the shock waves generated on both sides of the leading edge, when separation occurs, cannot cross supersonically. This problem is discussed in a later section.

The flow separation caused by the oblique shock on the wall and hub casings, although not analytically soluable, apparently should have little effect on compressor performance. The results of the supersonic inflow compressor test reported in reference 3 indicated predictable performance without measurable casing separation losses although evidence of such separation at an axial Mach number of 1.5 was noted by wall pressure measurements.

VARIABLE-PITCH GUIDE VANE ANALYSIS

The possibility that the relatively high stagnation pressure loss which invariably exists at the higher flight Mach numbers may be reduced through the use of supersonic guide vanes is investigated in this section. Such guide vanes not only diffuse the flow through the turning they produce, but can reduce the required supersonic flow deflection at the rotor leading edge, the major source of the loss. In a sense, the use of the guide vanes, hopefully, would be analogous to the use of a system of multiple oblique shocks in an inlet designed for high recovery. The vanes will be presumed to be uncambered and capable of angle-of-attack variation during flight to avoid inlet starting problems associated with internal area contraction.

The governing fluid dynamic equations are presented in Appendix B and a solution is obtained for the exiting parameters of flow angle, Mach number, and stagnation pressure recovery. For given inflow conditions, the assumed uniform discharge flow is found to be a function only of the vane loading σC_L and the drag-lift ratio. The use of arbitrarily assigned values of σC_L and C_D/C_L to demonstrate the effects of these parameters often leads to impossible solutions because the second law of thermodynamics is violated. Therefore the aerodynamic coefficients calculated by shock-expansion theory for a specific blade shape were used to demonstrate a solution to the guide vane equations. In the choice of solidity, guidance was obtained from the linearized two-dimensional theory of flat-plate cascades (ref. 7) which indicates that the mutual interference between blades will drastically alter the aerodynamic force acting on each blade. The linearized loading σC_L at a fixed angle of attack will be a maximum at that value of solidity for which the leading Mach wave from one blade intersects the trailing edge of the next blade and the loading then oscillates between this maximum value and zero as the solidity increases. The normal force loading σC_N calculated by the linearized theory for a flat-plate cascade is compared in figure 9 with that using the shock-expansion wave theory. The notation "discontinuous" in the legend refers to the fact that the pressure variation along the entire expansion region at the surface is assumed to change discontinuously across the median expansion wave. The term "shock expansion linear" refers to the fact that the pressure is assumed to vary linearly in two steps through the expansion region. It is apparent that the maximum loading for the more exact case (shock expansion-linear) occurs only when the guide vane chord length is chosen such that the shock impinges at the trailing edge. Further, the linearized theory maximum loading value is considerably in error. It therefore appears unnecessary to consider solidities greater than the maximum noninterference value for guide vane use. The maximum noninterference value of solidity can be found approximately from the following equation which assumes constant shock wave inclination angle (ref. 8):

$$\sigma_M = \frac{\cos \phi}{\sin (\phi - \alpha)}$$

The approximate maximum solidity is very nearly equal to the true value except where the flow behind the shock approaches the sonic value.

For the purpose of illustration of the maximum guide vane turning and diffusion that may be achieved, a 5-percent-thick, symmetrical diamond-profile section with an assumed average skin-friction coefficient per surface of 0.003 was chosen for examination. Figure 10 presents the maximum noninterference solidity and corresponding blade loading as a function of angle of attack and inflow Mach number. The aerodynamic coefficients were determined by shock-expansion theory for isolated airfoils and the angle of attack was varied in each case until sonic flow was reached behind the oblique shock. The calculated guide vane exiting flow parameters for these maximum noninterference solidities are presented in figure 11 for that solution of equation (12) of Appendix B which yields the lower entropy increase. Unlike the subsonic flow case, not only do the guide vanes diffuse the flow as may be anticipated, but what is surprising is that the flow turning angle β_2 is considerably greater than the angle of attack and the stagnation pressure loss can be prohibitively high. Exceeding the angle of attack for which the loading is a maximum results in reduced turning and a rapid increase in pressure loss with little additional diffusion.

The effects of varying the angle of attack of the supersonic guide vanes upon the inflow conditions to the compressor are shown in figure 12 for a flight Mach number of 3. The solidity was chosen to be 0.5 to preclude mutual aerodynamic interference within the vanes. Although increasing the guide vane angle of attack in a manner to produce turning against the direction of rotation did indeed reduce the oblique shock loss significantly, the net effect was to increase the overall maximum stagnation pressure recovery 2 percent above that obtained without guide vanes due to the additional guide vane loss. This increase in pressure recovery is accompanied by a 13-percent increase in surface Mach number. The use of guide vanes of twice the solidity (not shown) resulted in a further increase of 2 percent in the maximum stagnation pressure recovery. Even though the gains in stagnation pressure recovery with guide vanes are admittedly small, two additional factors should be discussed before eliminating the use of guide vanes. One is an examination of the flow in the blade passage behind the system of leading-edge shocks, the other is a consideration of the flow field immediately downstream of the guide vanes.

For the higher flight Mach numbers considered, the flow deflection produced at the compressor blade leading edge without the use of guide vanes is so large as to prevent either shock reflection from the pressure surface in the nonseparated leading-edge case (labeled inviscid) or the supersonic crossing of the oblique shocks for the separated leading-edge case. The flow patterns in figure 13 are illustrative of what occurs. In either case, a small indeterminant portion of the flow traverses a normal shock with its high attendant loss as shown by the curve labeled 2. It is not known if such a shock pattern is permissible although the schlieren photographs of reference 9 indicate similar patterns caused by back pressure do exist. Because a relatively small portion of the flow traverses the normal shock, the average relative stagnation pressure recovery across the passage will be considerably closer to the value of the curve labeled 1 which represents the loss through two oblique shocks than to the curve labeled 2. In the event that the flow patterns depicted are unstable, the compressor will force a bow shock to be positioned ahead of the cowl lip. The use of sufficient turning in the guide vanes will not only eliminate this problem, but, due to turning against the direction of rotation, will increase the relative stagnation pressure and consequently the work input ability of the compressor. At high flight Mach numbers the leading-edge separated-flow model as compared to the inviscid case exhibits the higher relative stagnation pressure because it avoids the extremely high pressure-surface Mach number associated with supersonic expansion about the leading edge.

The solution of the supersonic guide vane equations was predicated upon the assumption of the uniform discharge flow velocity. The desirability of minimizing the distance between the guide vanes and the rotor from weight considerations will prevent the attainment of the assumed uniform flow. An example of the calculated circumferential distribution of the local Mach number and flow direction one-half chord length downstream of a typical guide vane configuration is given in figure 14 with the superposition of the average values obtained from the guide vane equations. For solidities less than the maximum noninterference value, a region exists immediately downstream of the vane in which the flow is essentially equal in direction and Mach number with that upstream of the guide vanes. Thus the possibility of using guide vanes to reduce the flight Mach number at which a supersonic compressor designed for subsonic axial inflow velocity will permit supersonic inflow cannot be realized. Further, the use of guide vanes to eliminate the normal shock losses at the entrance to the rotor passage as presented in figure 13 will be effective only for portions of each revolution giving rise to an oscillating flow pattern. It is therefore questionable whether any significant benefits may accrue through the use of variable-pitch supersonic guide vanes.

OPERATIONAL CONSIDERATIONS

The previous analyses have been based upon an idealized two-dimensional cascade concept for supersonic flight in the stratosphere. It is of interest to consider the effects of operational conditions or requirements which differ from these assumptions.

Free-Stream Influences

The primary free-stream influence affecting the inflow conditions at a given Mach number is due to temperature variations which occur either by accelerating at altitudes below the stratosphere or operating in the airplane-disturbed flow field. Assuming the altitude to vary linearly from 25,000 to 35,000 feet as the flight Mach number is increased from 1.4 to 1.8, the effect of temperature variation is to increase the inlet starting Mach number from 1.35 to 1.45 for the case illustrated in figure 15. On the same figure, the effect of an 8° flow deflection created by a two-dimensional wing shock positioned ahead of the inlet, the loss of which is not charged to the propulsion system, is shown to significantly increase relative stagnation pressure recovery and decrease the relative Mach numbers, both of which are desirable for improved performance.

Compressor Location

The supersonic compressor was assumed to be located behind a loss-free constant-area passage which turned the flow parallel to the axis of rotation. Because, as mentioned earlier, the primary advantage of the supersonic inflow compressor is the weight savings associated with the elimination of the subsonic portion of the inlet diffuser, positioning the compressor at the cowl lip would appear desirable to maximize the weight savings. In addition, location of the compressor at the cowl lip could affect a reduction in maximum nacelle diameter with a subsequent reduction in nacelle drag. The effects on the inflow conditions as a result of locating the compressor at the cowl lip are shown in figure 16. The radial component of velocity, assumed equal to the cone surface value times the sine of cone angle, remains unchanged across the oblique shock. The lower axial Mach number and deflection angle required yields a higher Mach number, stagnation pressure recovery, and inlet starting Mach number than that of the axial inflow configuration for a straight radial leading edge. This in a sense is analogous to the use of aerodynamic leading-edge sweep. Further improvements in relative stagnation pressure recovery are possible, at the expense of higher passage Mach numbers, if the leading edges of the rotor blades are physically swept. Improved compressor performance may be possible by placing the leading edge of the rotor tip at the cowl lip, and sweeping the rotor blade such that the leading edge of the hub section is well forward of the plane of the cowl lip. Since radial spillage for starting supersonic flow in the compressor is possible, the internal contraction in the passage may be increased beyond the two-dimensional starting value to reduce the internal stagnation pressure loss. No attempt has been made to explore this possibility.

Supersonic Restart Capability

As with any supersonic inlet, it is necessary that the inlet be capable of restarting should the terminal shock inadvertently be expelled. Supersonic inlets of the all-external compression type, such as considered herein, exhibit automatic restart capability when the disturbance which caused the unstart is eliminated. It would appear that when operating in conjunction with a supersonic inflow compressor the automatic restart capability may not exist for not only may the compressor blade passage employ internal contraction but when operating in the supersonic inflow condition the relative stream tube entering the compressor blade passage is contracted through the oblique shock an amount considerably in excess of the permissible starting value. However, the following analysis indicates that automatic restart of the inlet-compressor combination is possible and the conditions required for restart.

If for a given flight condition for which the leading-edge shock would normally fall within the blade passage, the vectorial sum of rotational velocity (assumed unchanged) and the velocity behind the normal shock on the cone during an inadvertent unstart always produces a relative air inlet angle greater than that of the "entrance region" of the blade. Thus for a flat-plate cascade expansion waves are produced (ref. 1) which act to accelerate the axial inflow velocity causing the shock to reenter the inlet and attach itself to the blade leading edge, if the disturbance which created the unstart no longer exists. At high flight Mach numbers, the relative inlet velocity to the compressor during the period of the unstart is close to the sonic value because of the elevated temperature. An unstarted wave pattern then exists at the entrance to the compressor with finite blade thickness as described in reference 1 either due to excessive internal contraction or leading-edge angle or both. If the Mach number behind the normal shock on the cone during the unstart is less than that entering the compressor in a static compressor test facility at a corrected rotational speed corresponding to the flight condition being investigated, the wave pattern at the compressor entrance is incompatible with the conditions existing behind the cone normal shock and will therefore cause the normal shock to reenter the inlet automatically, thereby restarting the supersonic inflow to the compressor. The same analysis may be employed to determine the flight Mach number required for the start of supersonic inflow to a compressor having a finite leading-edge radius.

Low-Speed Operation

The use of a sharp leading-edge inlet at subsonic flight speeds will create sizable inlet stagnation pressure losses in a manner described in reference 10. In the present case, the combination of a sharp cowl lip, a constant area passage, and a sonic inflow requirement to the compressor will produce exceptionally high inlet stagnation pressure loss and a correspondingly large reduction of engine airflow at low subsonic flight Mach numbers. For example, at the start of the take-off roll, the maximum stagnation pressure recovery and airflow rate will be 79 percent of that obtainable with a well-rounded lip. Therefore some form of variable geometry such as inflatable cowl lip, blow-in-door, or translating cowl will be required to prevent the very significant thrust decrement represented by this loss. If supersonic guide vanes are included in the design, their use in reducing the inflow velocity during take-off by turning with the direction of rotation would be ineffective in reducing the loss because of the relative insensitivity of the loss to the inlet Mach number variation achievable. The location of the compressor at the cowl lip may eliminate a large portion of the low-speed loss due to the additional energy added to the flow by the compressor in the region of separation and mixing; however, this is pure conjecture as the problem is not amenable to analysis and experimental evidence is lacking.

Creation of Sonic Inflow

In the foregoing, it was assumed tacitly that sonic inflow velocity to the compressor is realizable. Ideally, in the absence of viscosity and with sharp leading-edge blades (in the mathematical sense), it is possible to induce sonic inflow immediately upstream of the compressor. Practically, however, the finite leading-edge thickness and the effects of viscosity in rapidly forming a boundary layer at the leading edge will not permit the achievement of the desired inflow velocity. Reference 11, which reports the highest known inflow velocity achieved to date, namely, 0.82, discusses this effect in more detail. Generally, supersonic compressors designed for 0.8 axial inflow Mach number and utilizing a straight "entrance region" operate with approximately 2° positive angle of attack with respect to the suction surface indicating that some form of wave pattern exists upstream of the rotor. Shadowgraphs showing the existence of such waves are contained in reference 12. The effect of the boundary-layer buildup can be circumvented by setting the blade suction surface at a lower angle than that for ideal sonic entry, but the finite leading-edge thickness which exists even for "sharp" blades will create waves which must stand ahead of the plane of the leading edges. It is felt that the upper subsonic inflow Mach number that could be achieved is approximately 0.95. Thus the inlet starting flight Mach number will be slightly higher than the values indicated previously; this, however, will have no appreciable effect on the spillage requirement.

Engine Performance Implications

A cursory investigation was made of the effects of the supersonic flow induction process upon the propulsion system performance as compared with that using a conventional subsonic entry compressor. For this comparison two turbofan designs were chosen arbitrarily, each of which had a design fan pressure ratio of 2, overall pressure ratio of 9, and a bypass ratio of 1. The engines were assumed to have the same internal component efficiencies with the exception of the fan stage, the same internal pressure losses, and to burn stoichiometrically in the combustor and fan duct. The transonic fan efficiency was chosen to be 85 percent, the sonic inflow fan to be 75 percent. The inlets were sized to capture the same airflow rate at a flight Mach number of 3. The pressure recovery of the conventional inlet was presumed to follow the military specification 5008B pressure recovery schedule without boundary-layer bleed.

At a Mach number of 1.3 which is close to the minimum thrust-minus-drag value of the aircraft, the stagnation temperature is equal to the sea-level standard value and the engines are operating at their design value of pressure ratio. The net internal specific thrust, and consequently specific impulse, are essentially equal for both types of engines. The engine airflow rate of the supersonic inflow engine in conjunction with a 20° cone, as noted previously, is 13 percent greater than that of the subsonic entry engine and for the same conical angle the external spillage drag associated with supercritical operation in each case is 20 percent less (ref. 5) all of which represent a sizable increase in the net accelerating force. At a flight Mach number of 3, assuming the same work input for each element of the compressors, the maximum specific thrust of the supersonic inflow engine is only approximately 2 percent less than that using the subsonic inflow compressor even though the supersonic compressor efficiency falls to approximately 12 percent as a result of charging the compressor with the additional losses associated with the compressor passage shock system assuming flow separation at the leading edge. Although not subjected to analysis, it would appear that the maximum net installed specific impulse at cruise thrust for a Mach number of approximately 2 would be equal for both engines but at a flight Mach number of 3, the cruise performance of the supersonic inflow engine would be prohibitively inferior to that using the subsonic compressor.

It should be pointed out that the supersonic inflow engine was assumed to be flow-matched in all components. Establishment of component and therefore overall engine performance for matched operation is not possible in the absence of performance data for supersonic compressors in the supersonic entry mode of operation. With the latitude afforded by variable area turbines and exit nozzles, matching is expected to be achieved without affecting the conclusions reached above.

Variable Geometry Inlet.

The high compressor inlet losses noted at flight Mach numbers near 3, which are caused by the large supersonic flow deflections required at the blade leading edge, may be reduced through the use of a variable geometry cowl design. The use of an iris-type expanding-diameter cowl lip creating internal contraction, as suggested in reference 2, will reduce the compressor inlet Mach number and thus the strong oblique shock loss. The additional benefits of increasing the engine airflow rate or conversely reducing the engine size for a given thrust, plus a reduction of the low-speed-sharp-inlet-loss due to the variable geometry feature must be tempered against the increased complexity, nonautomatic restarting cycle, and the additional weight associated with increased distance from the cowl lip to compressor to permit shock reflection. In addition, the degree of internal contraction is limited because the throat area is fixed by the compressor, and increases in cowl lip diameter to values much greater than the maximum nacelle diameter to provide adequate area contraction will create additional internal shock losses and external drag. Although performance gains may be achieved through the use of variable geometry it is doubtful that cruise propulsion efficiencies equivalent to that of the conventional-type engine at a flight Mach number of 3 are possible.

CONCLUDING REMARKS

The flow induction process of compressors designed to accept supersonic inflow velocity has been examined. The limiting conditions which ensure supersonic inflow have been delineated indicating the desirability of designing the compressor to induce nearly sonic inflow velocity at sea-level standard conditions. At high flight Mach numbers, approaching 3, the high-leading-edge stagnation pressure loss must be considered in the analysis of any propulsion system employing the supersonic compressor. The capability of the supersonic inflow engine to produce a significant increase in thrust in the transonic accelerating flight range at the probable expense of poorer fuel economy at Mach 3 cruise conditions makes its use attractive primarily for hypersonic accelerator engines.

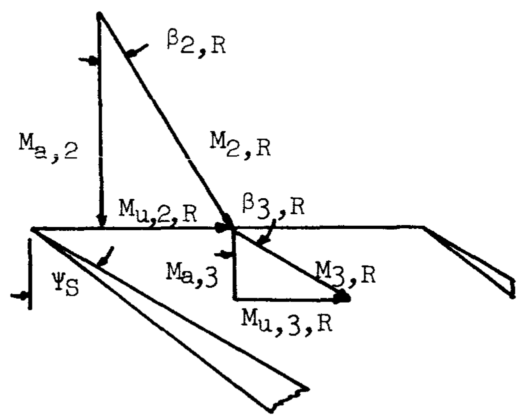
APPENDIX A

CALCULATION OF THE LIMITING CONDITIONS FOR SUPERSONIC INFLOW OPERATION

The limiting condition for supersonic inflow exists when the oblique shock spans the cascade entrance as in the sketch. Assuming values for $M_{2,R}$ and $\beta_{2,R}$, $M_{a,3}$ can be found from gas dynamic tables as the Mach number behind a normal shock for an upstream Mach number equal to $M_{a,2}$ $\left(= M_{2,R} \cos \beta_{2,R}\right)$. The tangential velocity is unchanged such that the downstream tangential Mach number is given by

$$M_{u,3,R} = M_{u,2,R}\sqrt{T_3/T_2}$$

$$= M_{2,R} \sin \beta_{2,R}\sqrt{T_3/T_2}$$



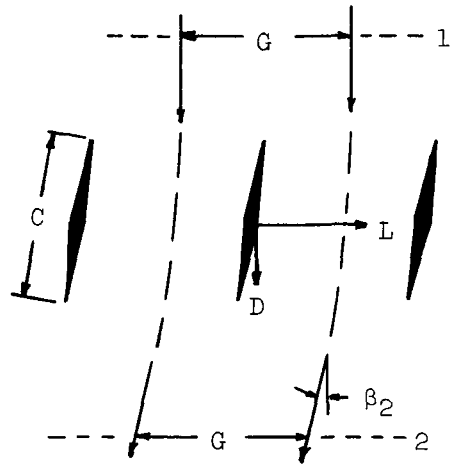
where T_3/T_2 corresponds to the static temperature ratio across the shock, therefore

$$\beta_{3,R} = \psi_S = \arctan \frac{M_{u,3,R}}{M_{a,3}}$$

APPENDIX B

DERIVATION OF COMPRESSIBLE FLOW GUIDE VANE EQUATIONS

It shall be assumed that the guide vanes turn the incoming flow, adiabatically, from uniform upstream conditions to a downstream location where uniform flow has been reestablished, as shown in the sketch. Wall friction will be neglected; however, frictional drag forces on the guide vanes are included. The air is treated as a perfect gas with constant specific heat. The appropriate equations are:



Continuity

$$\rho_1 V_1 = \rho_2 V_{a,2} = \rho_2 V_2 \cos \beta_2 \tag{1}$$

Momentum

Axial
$$p_1 + \rho_1 V_1^2 = p_2 + \rho_2 V_2^2 \cos^2 \beta_2 + D/G \tag{2}$$

Tangential
$$L/G = \rho_2 V_2^2 \cos \beta_2 \sin \beta_2 \tag{3}$$

Energy

$$C_p T_1 + \frac{V_1^2}{2} = C_p T_2 + \frac{V_2^2}{2} \tag{4}$$

Entropy

$$S_2 - S_1 \geqq 0$$

Plus the usual definition of solidity, $\sigma = c/G$; Mach number V/a; lift coefficient, $C_L = L\Big/\frac{1}{2}\rho_1 V_1^2 c$; and drag coefficient $C_D = D\Big/\frac{1}{2}\rho_1 V_1^2 c$. Equations (2) and (3) can be written explicitly for p_2/p_1 using the definitions of Mach number, solidity, lift and drag coefficients as

$$p_2/p_1 = \frac{1 + \gamma M_1{}^2\left(1 - \frac{\sigma C_D}{2}\right)}{1 + \gamma M_2{}^2\cos^2\beta_2} \tag{5}$$

$$p_2/p_1 = \frac{\sigma C_L M_1{}^2}{2M_2{}^2\cos\,\beta_2\,\sin\,\beta_2} \tag{6}$$

The static pressure ratio is eliminated by combining (5) and (6)

$$\frac{1}{M_2{}^2} = \cos\,\beta_2\,\sin\,\beta_2\left[\frac{2}{\sigma C_L}\left(\frac{1 + \gamma M_1{}^2}{M_1{}^2}\right) - \gamma\frac{C_D}{C_L}\right] - \gamma\,\cos^2\beta_2 \tag{7}$$

The continuity equation (1) is transformed to:

$$p_2/p_1 = \frac{M_1}{M_2\,\cos\,\beta_2}\sqrt{\frac{T_2/T_T}{T_1/T_T}} \tag{8}$$

and from the energy equation (4):

$$T/T_T = \left(1 + \frac{\gamma - 1}{2}M^2\right)^{-1}$$

such that:

$$p_2/p_1 = \frac{M_1}{M_2\,\cos\,\beta_2}\sqrt{\frac{2 + (\gamma - 1)M_1{}^2}{2 + (\gamma - 1)M_2{}^2}} \tag{9}$$

Combining equations (6) and (9) results in:

$$\frac{1}{M_2{}^2} = \frac{2\,\sin^2\beta_2}{(\sigma C_L)^2}\left[\frac{2 + (\gamma - 1)M_1{}^2}{M_1{}^2}\right] - \frac{\gamma - 1}{2} \tag{10}$$

Eliminating M_2 by combining equations (7) and (10) results in:

$$\cos\,\beta_2\,\sin\,\beta_2\left[\frac{2}{\sigma C_L}\left(\frac{1 + \gamma M_1{}^2}{M_1{}^2}\right) - \frac{\gamma C_D}{C_L}\right] + \sin^2\beta_2\left\{\gamma - \frac{2}{(\sigma C_L)^2}\left[\frac{2 + (\gamma - 1)M_1{}^2}{M_1{}^2}\right]\right\} = \frac{\gamma + 1}{2} \tag{11}$$

if by definition $B = \frac{2}{\sigma C_L}\left(\frac{1 + \gamma M_1{}^2}{M_1{}^2}\right) - \frac{\gamma C_D}{C_L}$ and $C = \gamma - \frac{2}{(\sigma C_L)^2}\left[\frac{2 + (\gamma - 1)M_1{}^2}{M_1{}^2}\right]$

then the equation can be solved explicitly for β_2 in the form:

$$\sin^2\beta_2 = \frac{B^2 + (\gamma + 1)C}{2\left(B^2 + C^2\right)} \pm \frac{\sqrt{\left[B + (\gamma + 1)C\right]^2 - (\gamma + 1)^2\left(B^2 + C^2\right)}}{2\left(B^2 + C^2\right)} \tag{12}$$

and

$$M_2 = \sqrt{\frac{1}{(\gamma - C)\sin^2\beta_2 - \left(\frac{\gamma - 1}{2}\right)}} \tag{13}$$

and

$$P_{T_2}/P_{T_1} = \frac{M_1}{M_2 \cos \beta_2}\left[\frac{2 + (\gamma - 1)M_2^2}{2 + (\gamma - 1)M_1^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \tag{14}$$

Note: For the case where $C_L = 0$ equations 12 to 14 are indeterminant. The exit flow parameters may be found from the analysis of reference 8 if $\frac{\sigma C_D}{2} M_1^2$ is substituted for C_F as defined therein.

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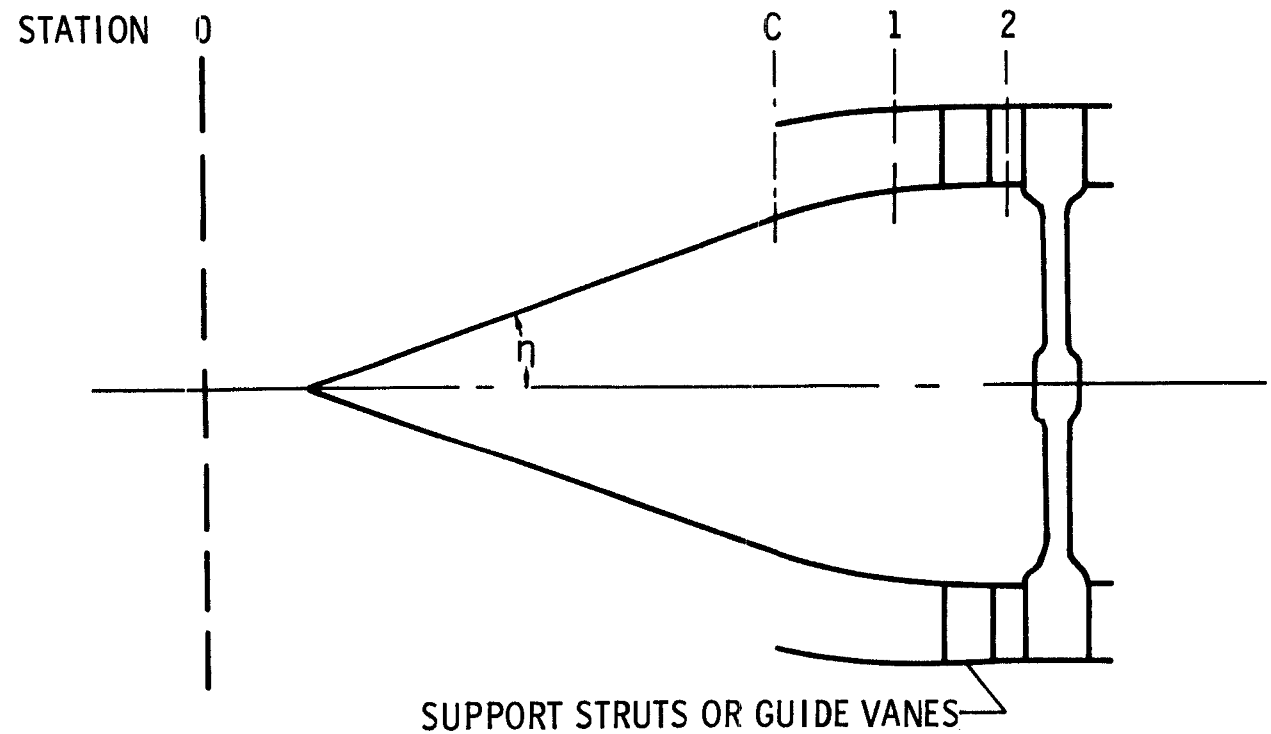

Figure 1.- Schematic diagram of inlet-compressor combination.

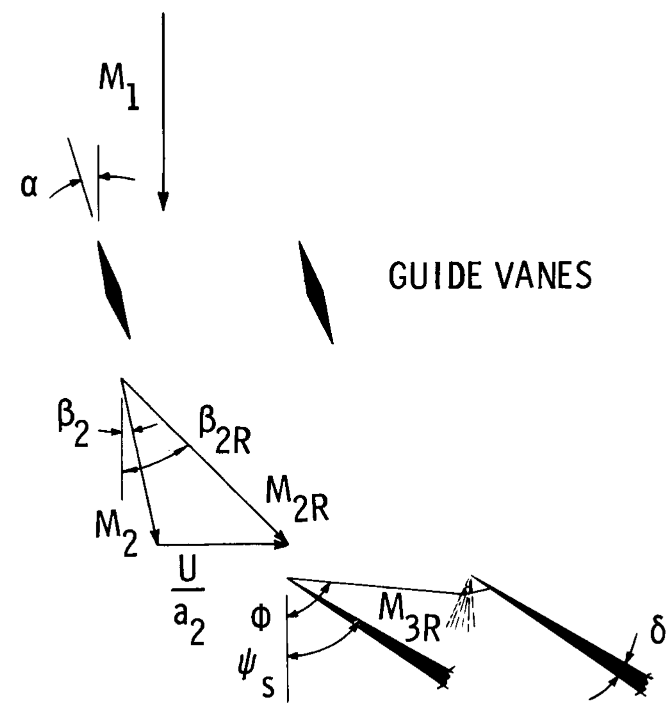

Figure 2.- Velocity diagram and inflow pattern to compressor.

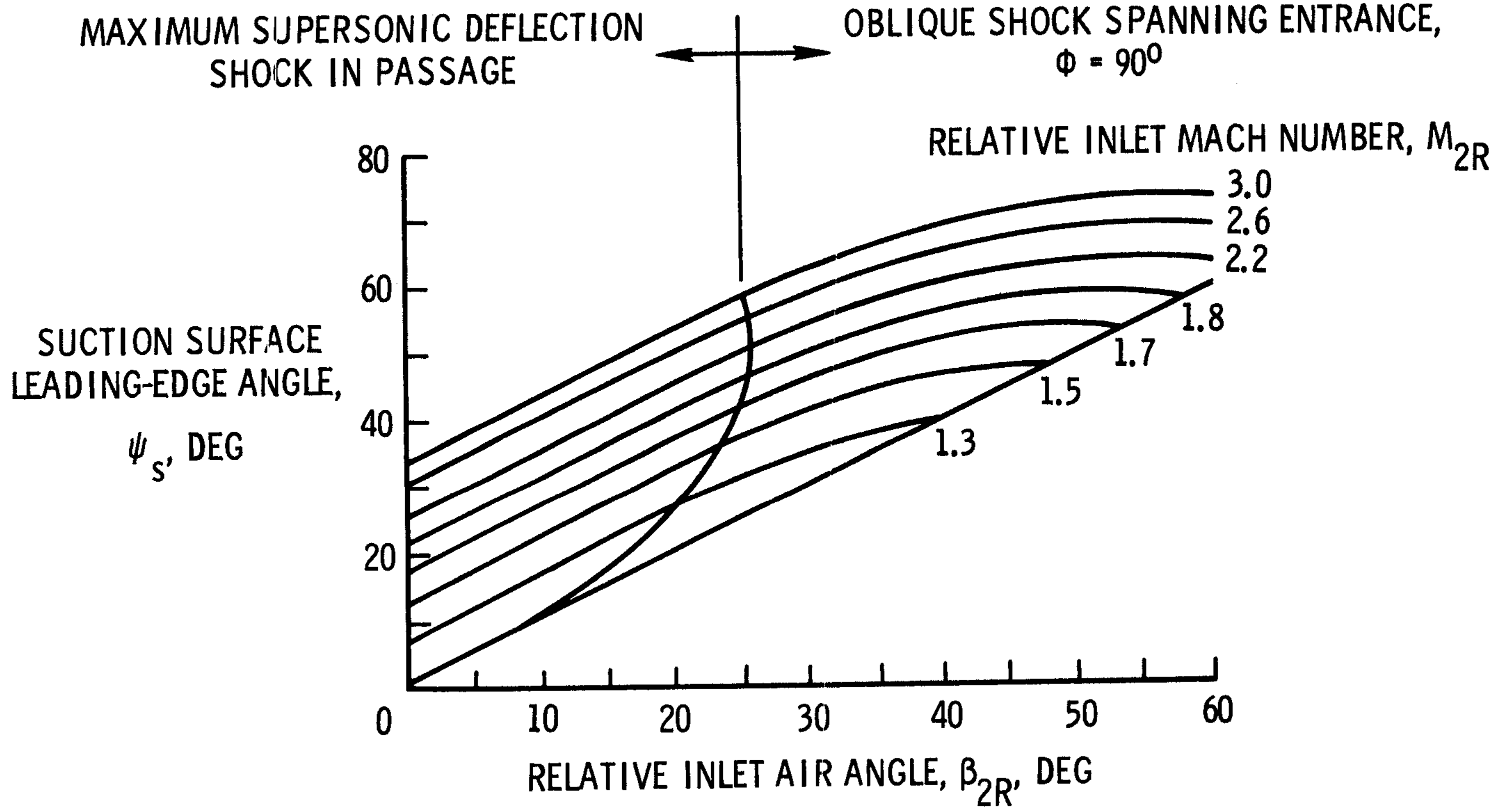

Figure 3.- Limiting conditions for valid supersonic inflow operation.

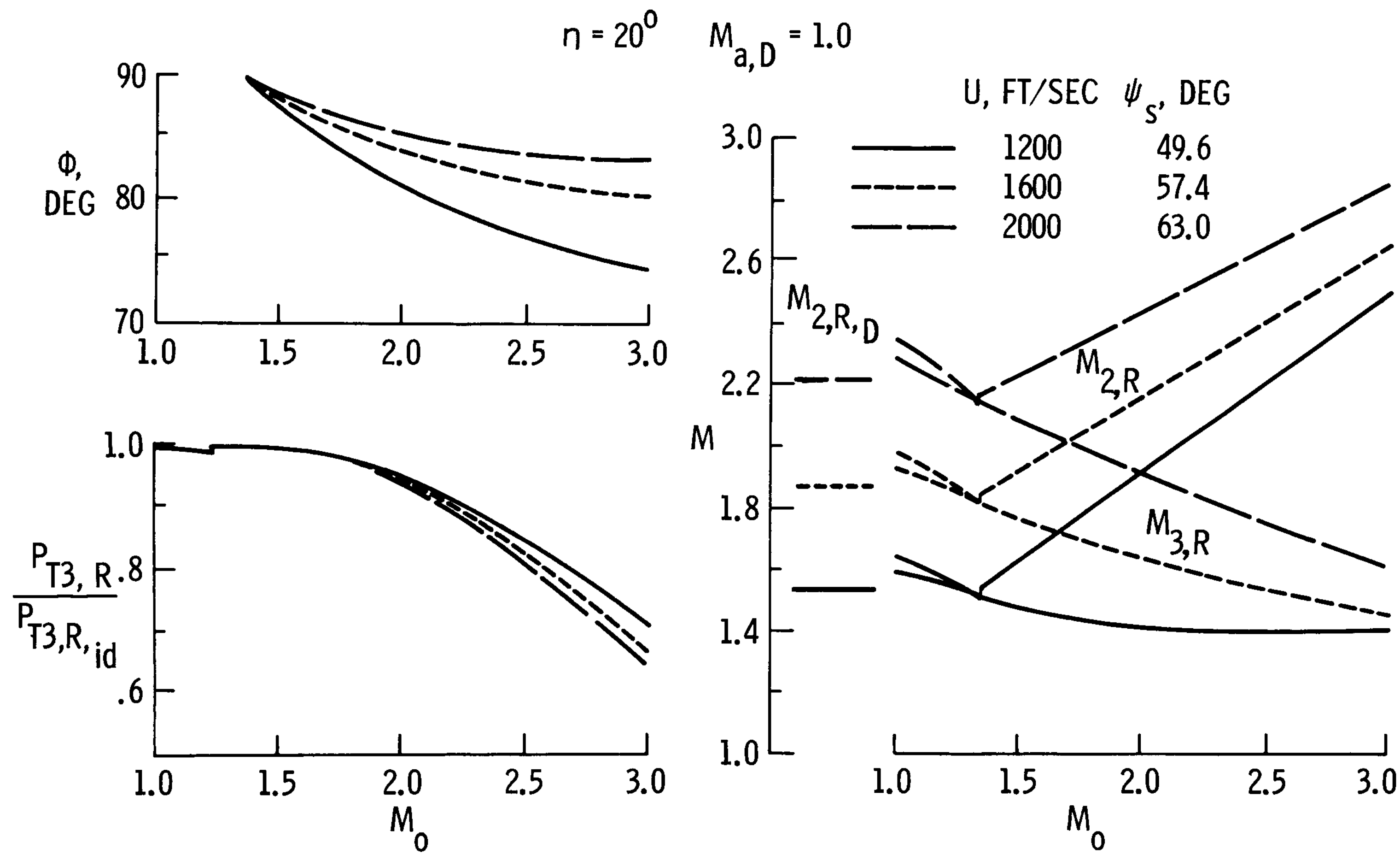

Figure 4.- Effect of rotational velocity upon inflow conditions.

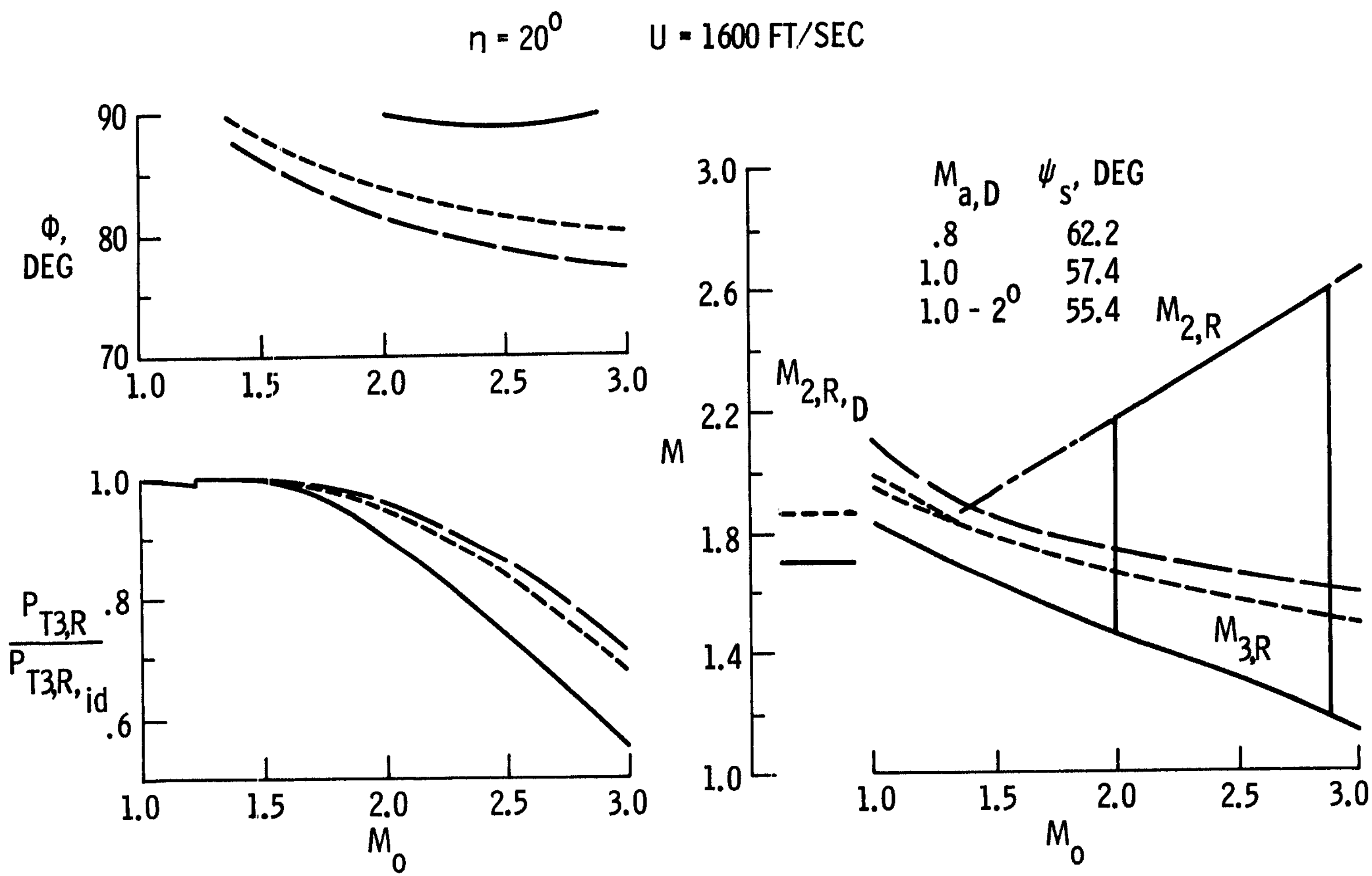

Figure 5.- Effect of blade surface angle upon inflow conditions.

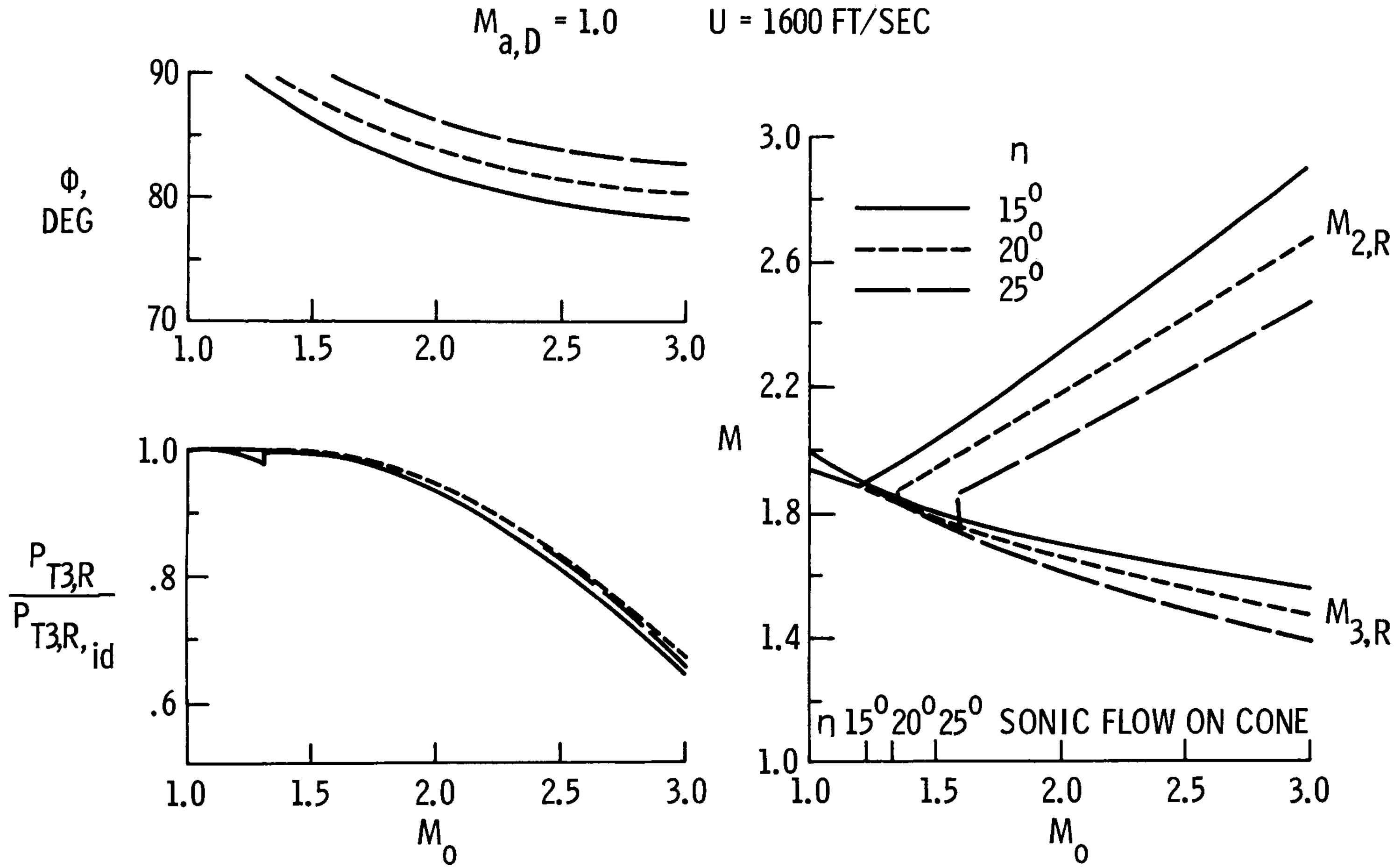

Figure 6.- Effect of inlet conical angle upon inflow conditions.

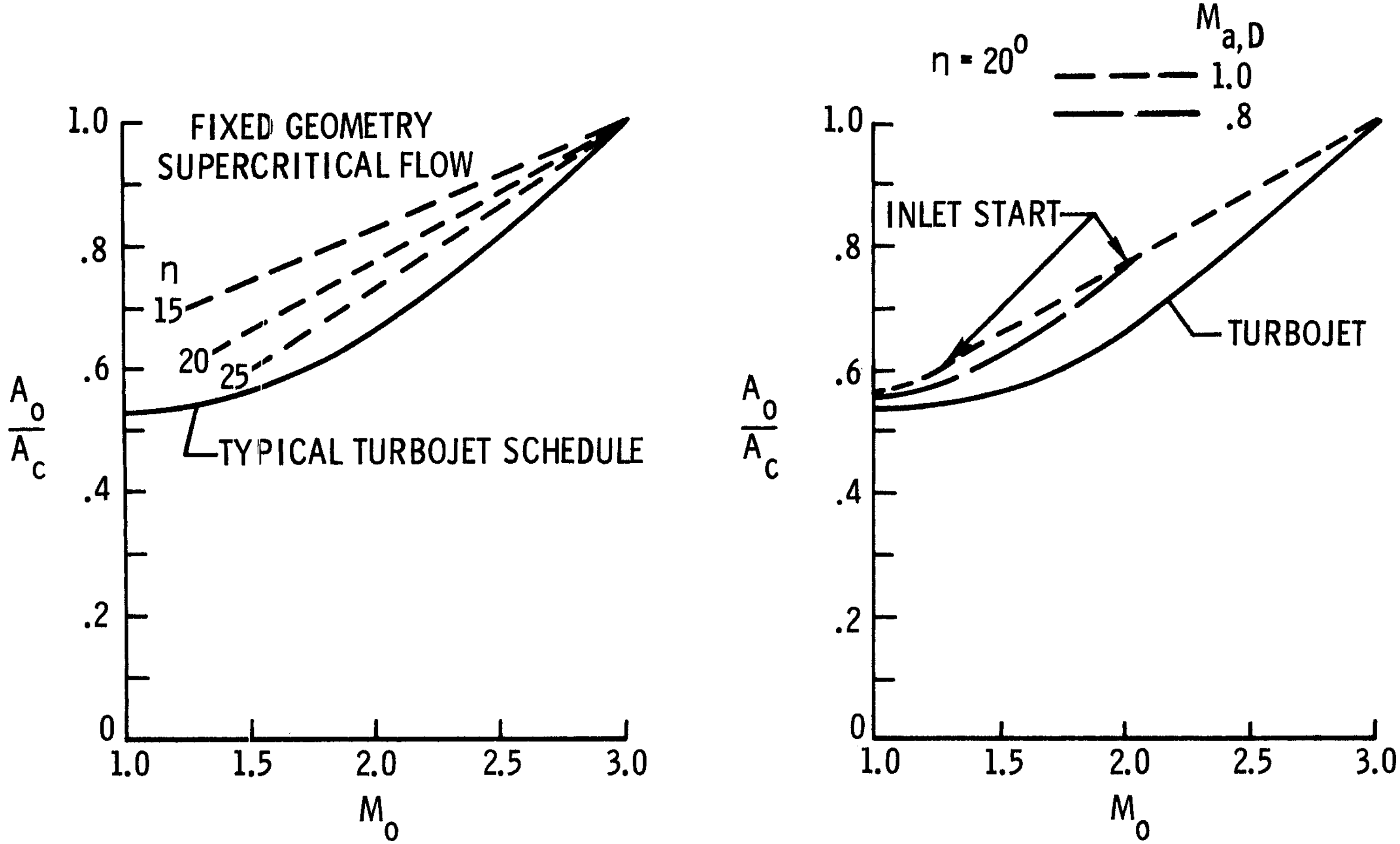

Figure 7.- Comparison of free-stream capture to cowl-lip-area ratio.

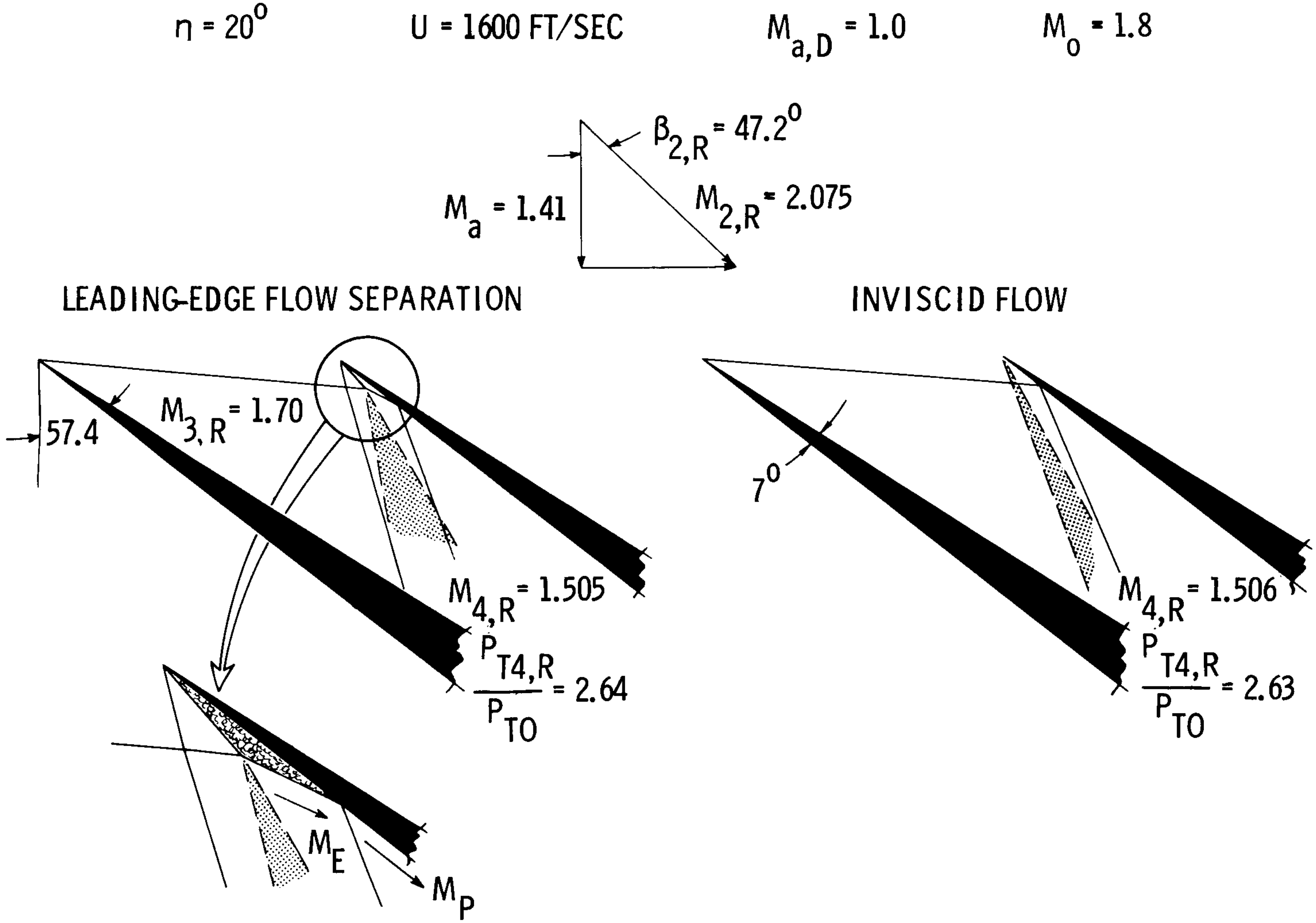

Figure 8.- Compressor inlet-flow pattern.

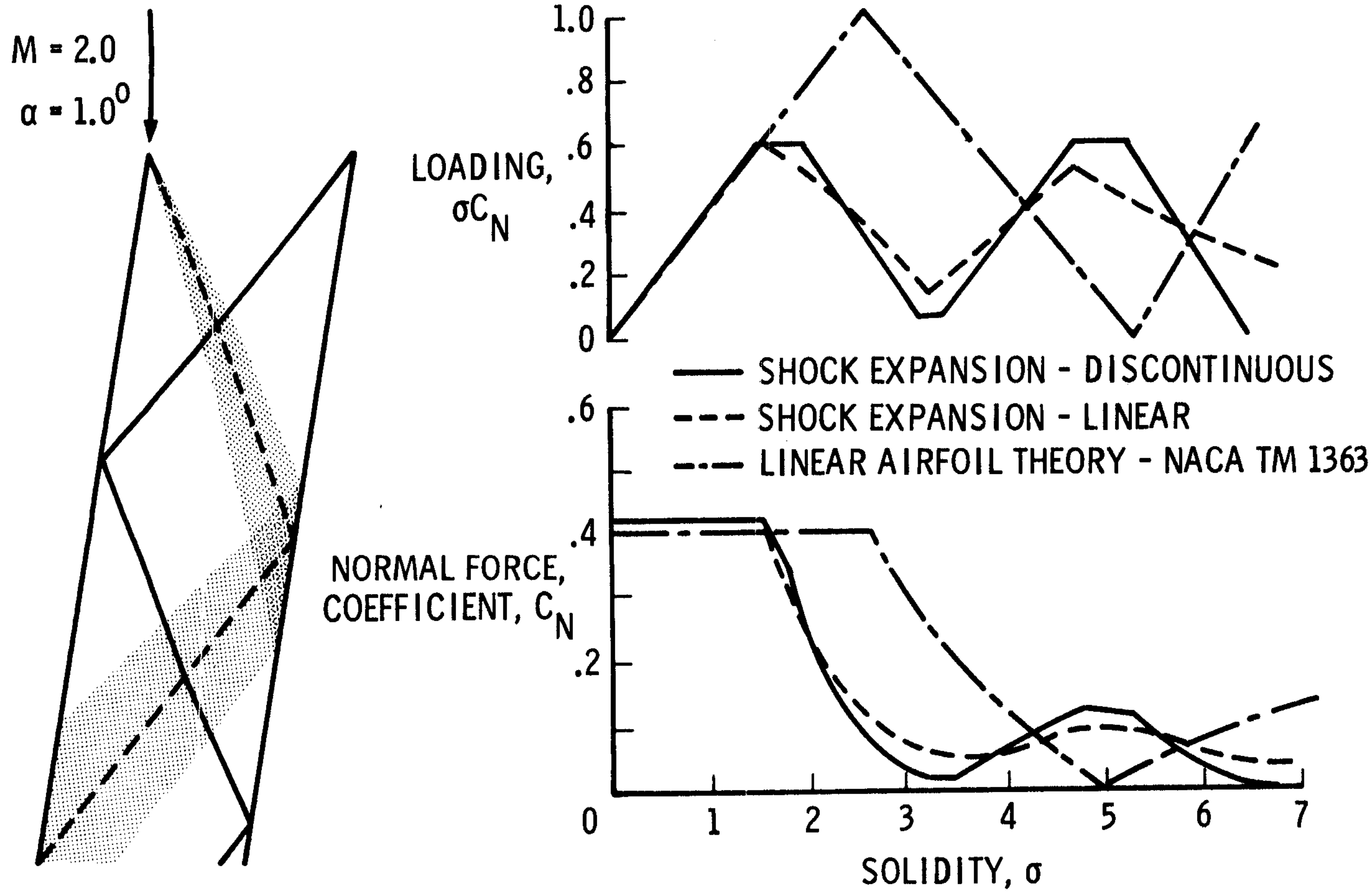

Figure 9.- Effect of solidity on flat plate blade loading.

SYMMETRICAL DIAMOND PROFILE, $t/c = 0.05$, $C_F = 0.003$

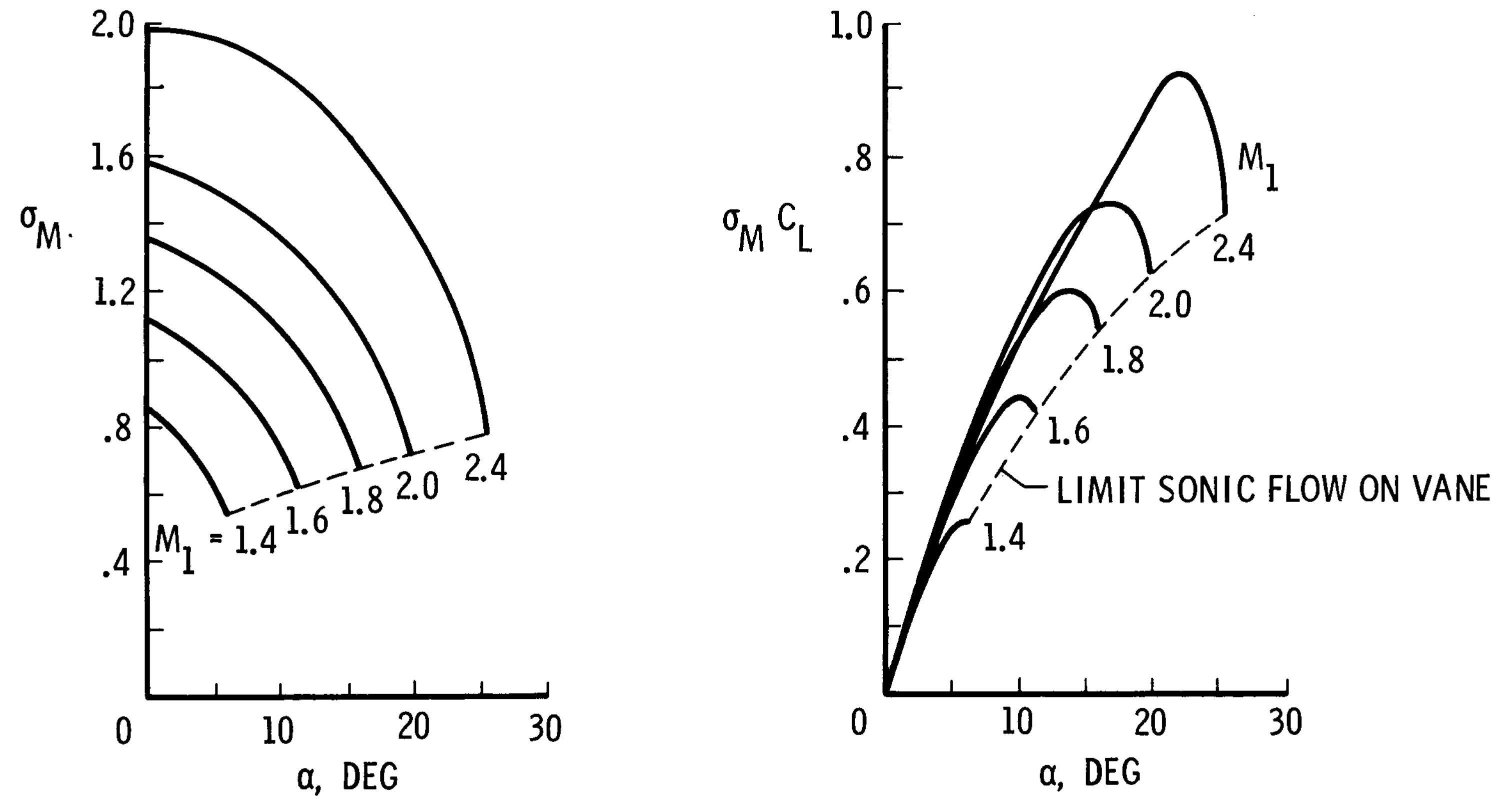

Figure 10.- Maximum solidity and blade loading of guide vanes without mutual interference.

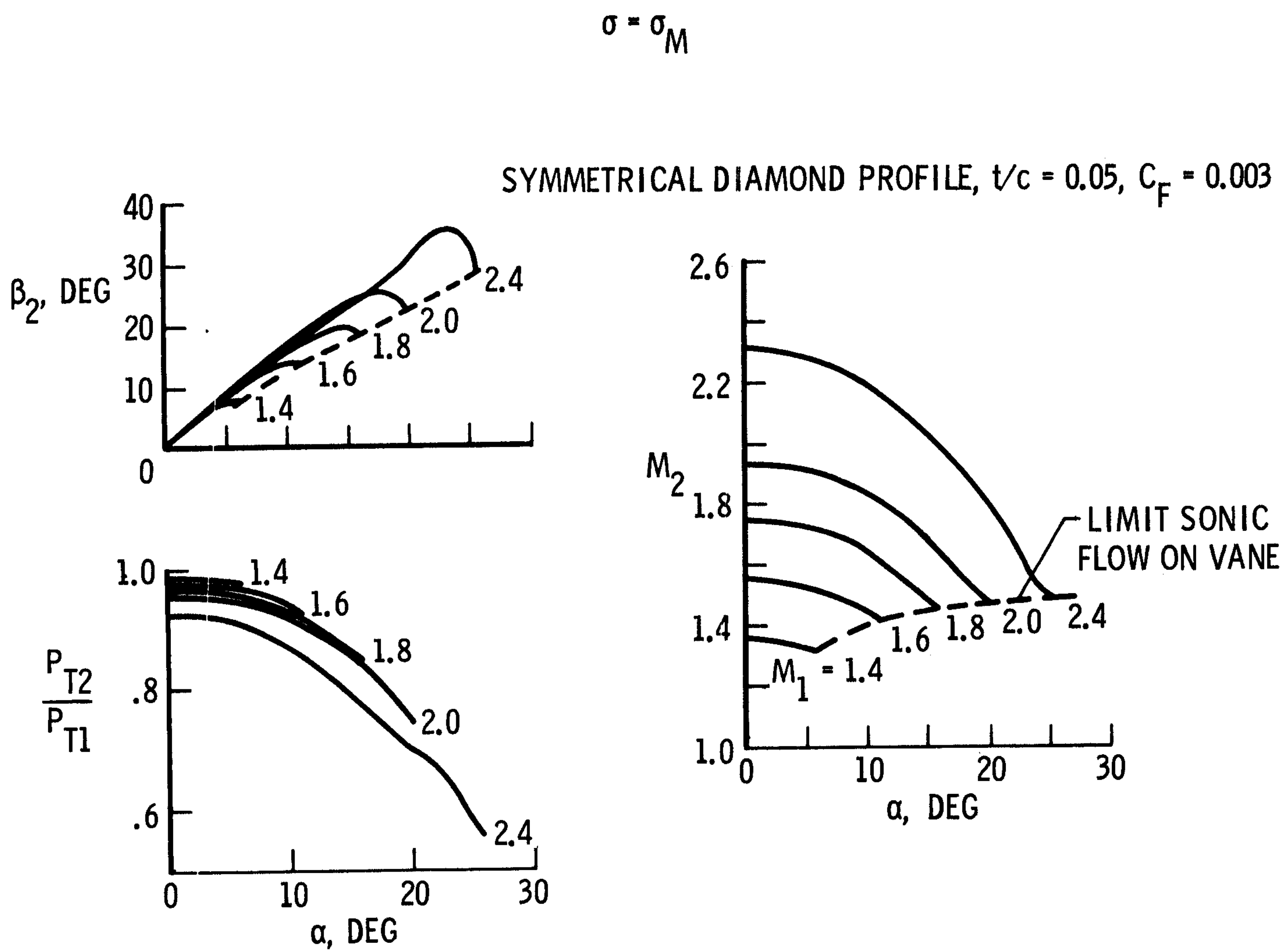

Figure 11.- Uncambered guide vane exit flow parameters.

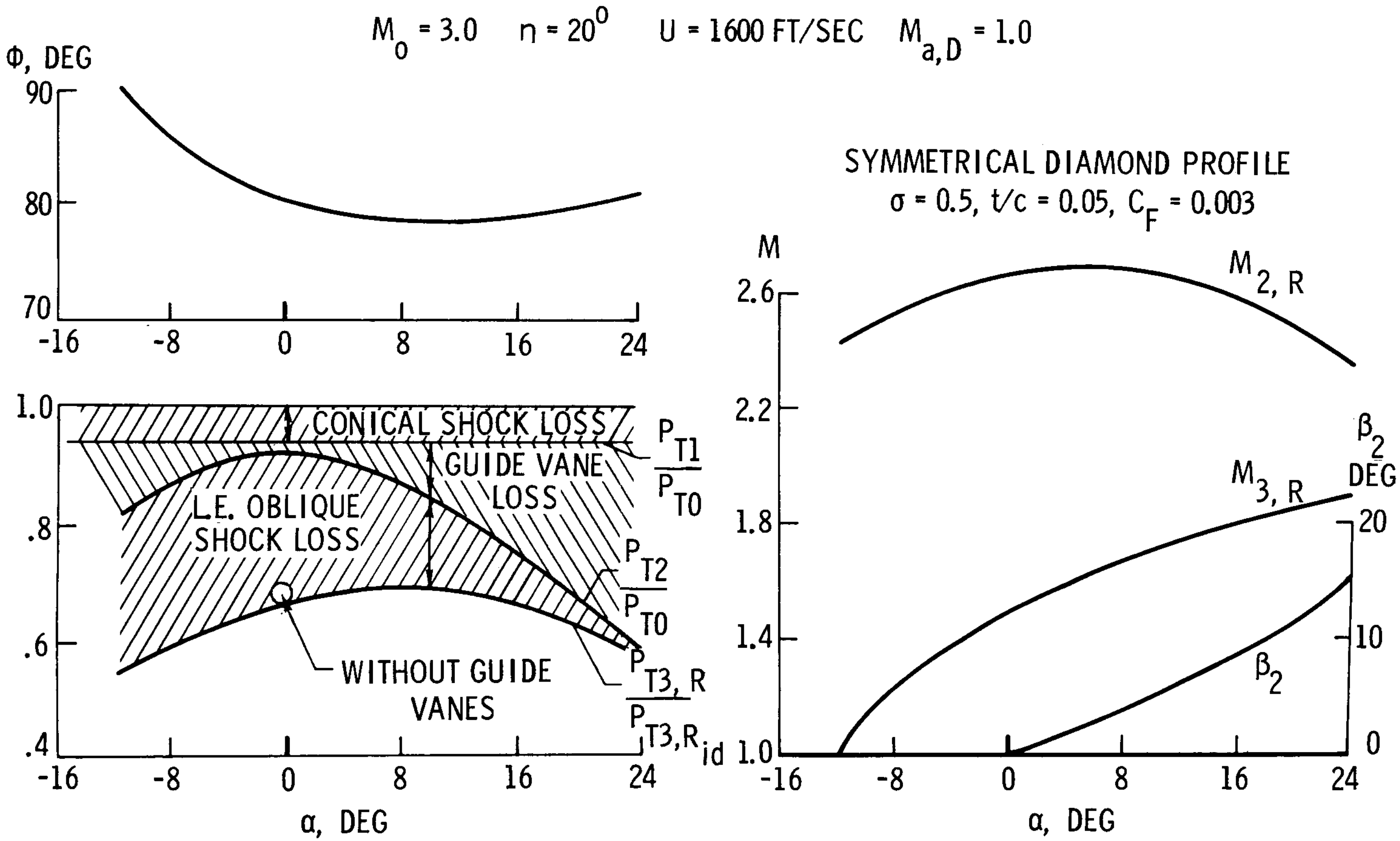

Figure 12.- Effect of guide vanes on compressor inlet conditions.

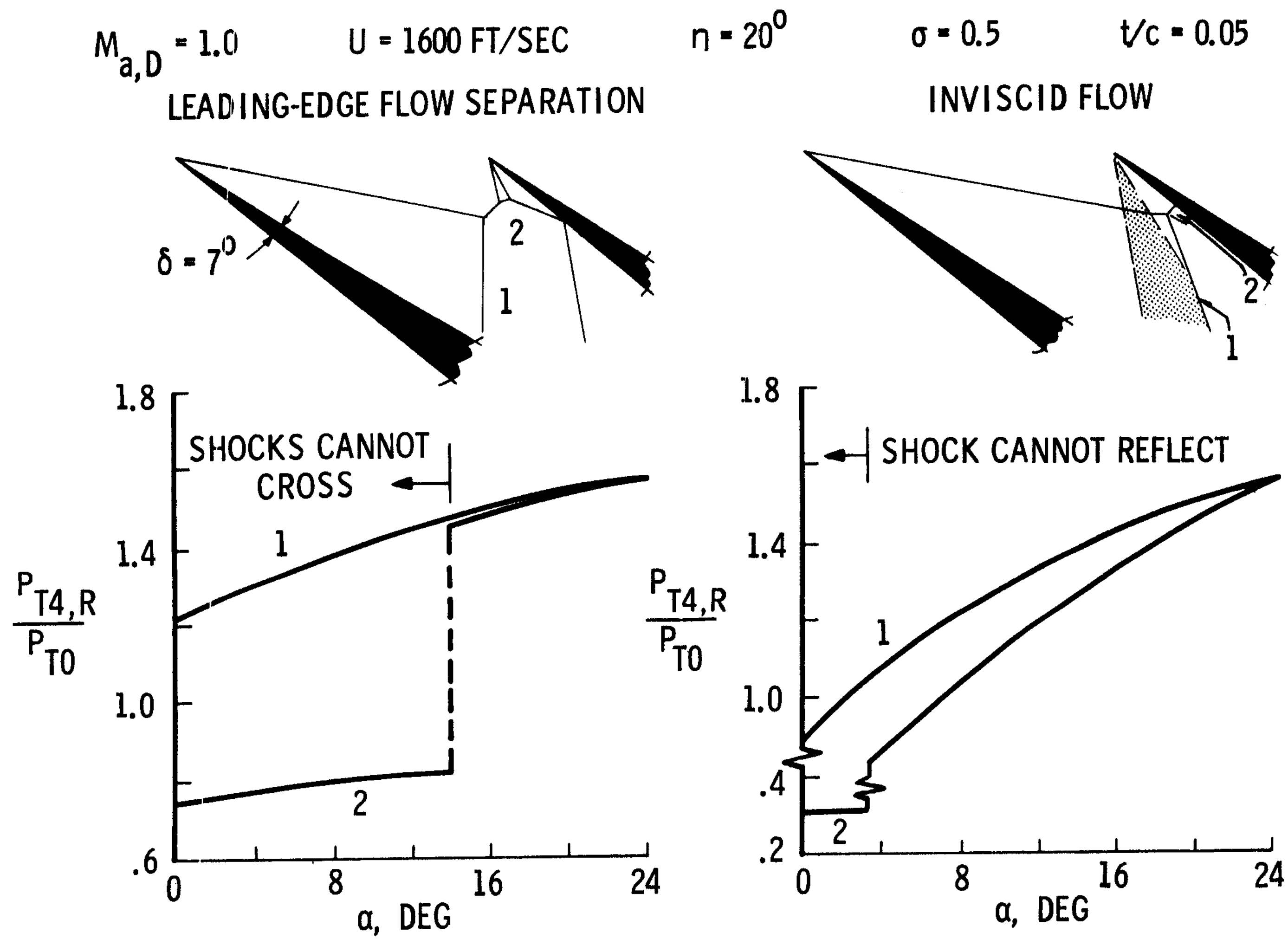

Figure 13.- Relative total pressure ratio in passage as a function of guide vane angle.

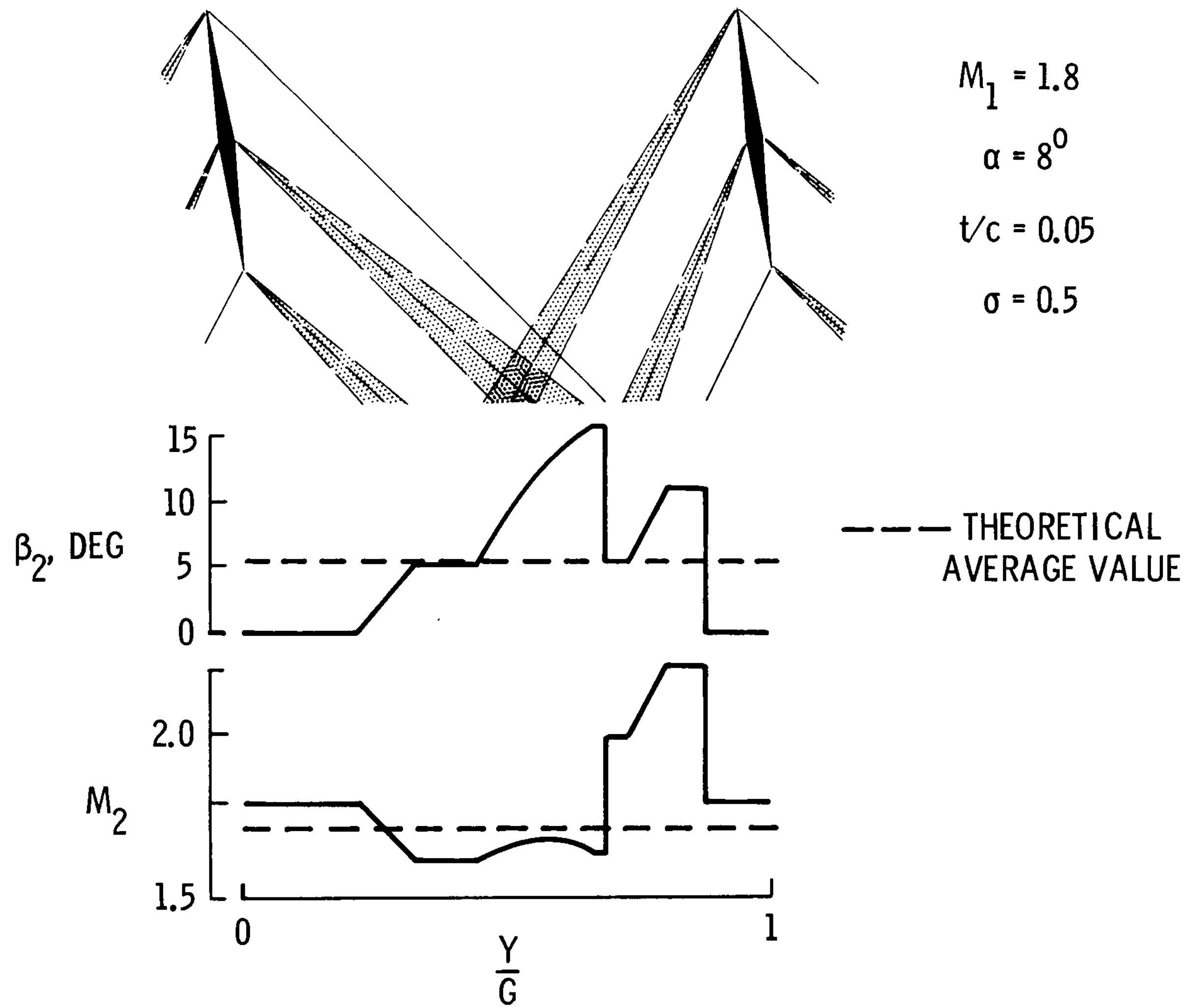

Figure 14.- Guide vane discharge flow pattern.

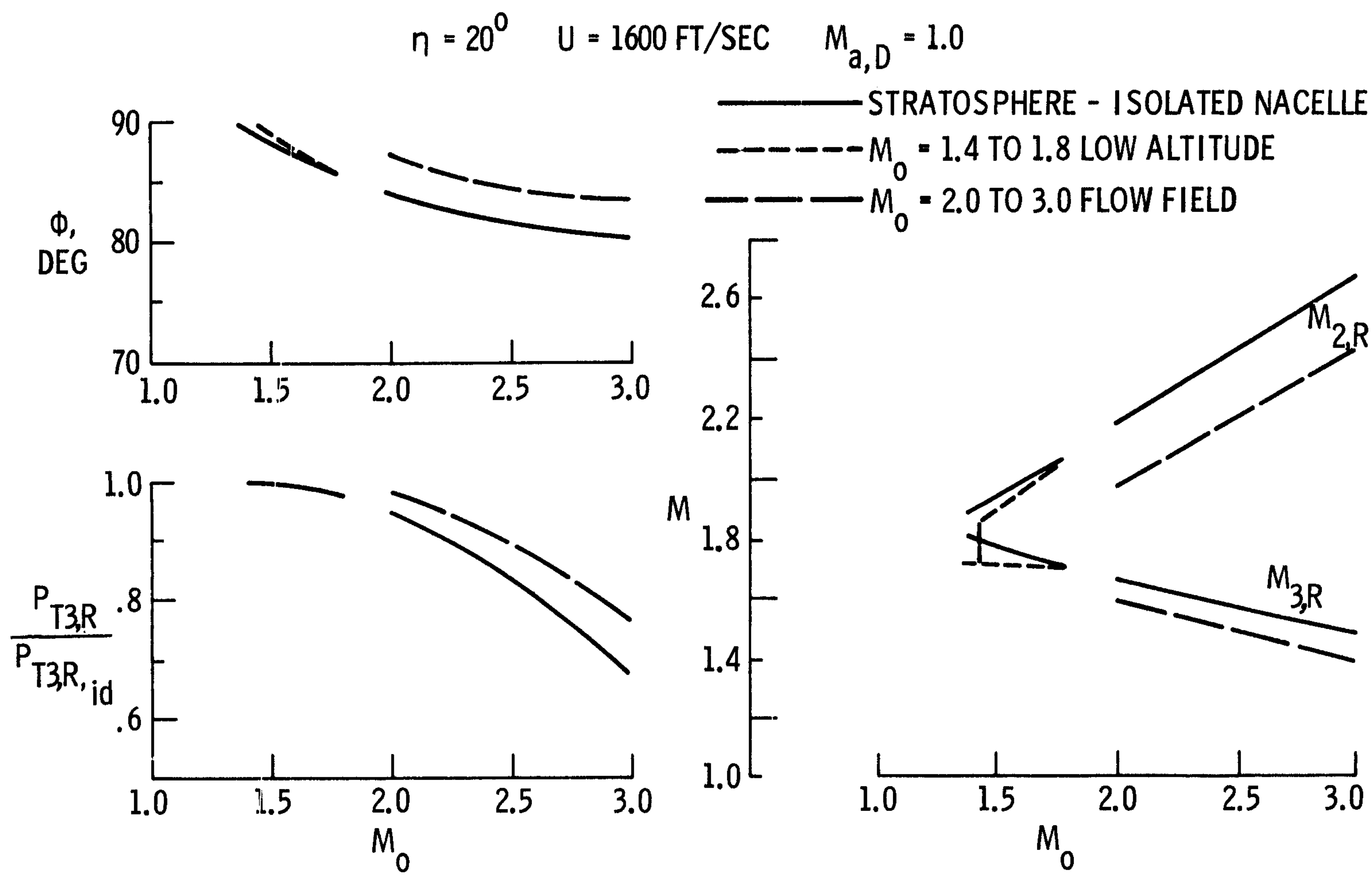

Figure 15.- Effect of altitude and airplane flow field upon inflow conditions.

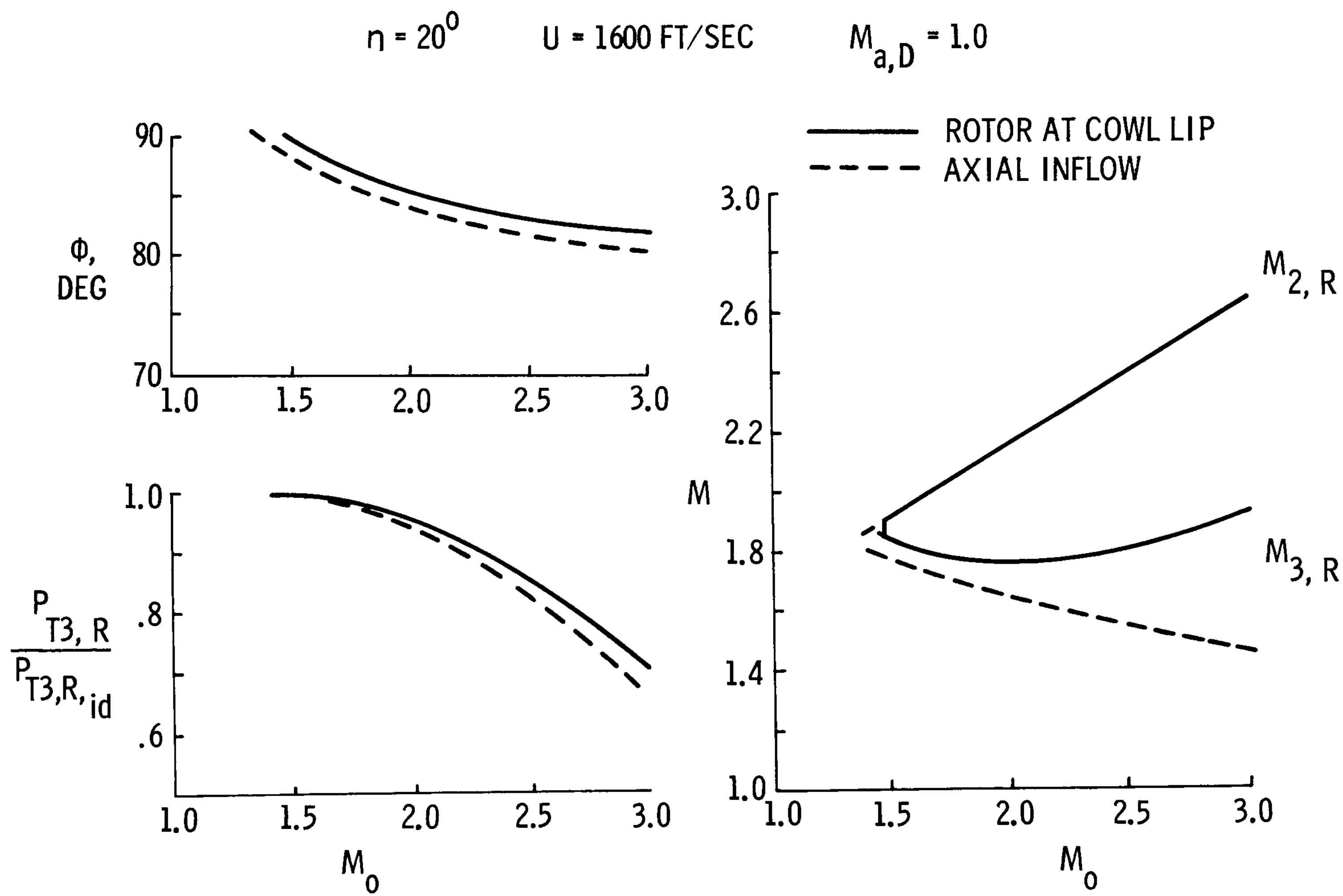

Figure 16.- Effect of locating compressor at cowl lip.

NASA-Langley, 1966